Light scattering from a quantum degenerate bosonic atomic gas

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Preliminary for slide 1

First of all I would like to thank the organizers for inviting us and for the opportunity to present our results at so nice conference. Today I would like to talk about “Light scattering from a quantum degenerate bosonic gas”. This is part of our entire theoretical investigations of light scattering from cold and ultracold atomic systems, which we do collaboratively with our experimental partners – Sergey Kulik (MSU), Julien Laurat (LKB) and Mark Havey (ODU).

Content:

- Motivation: Light - matter interaction in conditions of quantum degeneracy
- Overview of experiments on light scattering from BEC
- Microscopic theory of a single-photon scattering from a degenerate bosonic gas
- Results: Scattering spectra from BEC samples with different profiles of the order parameter
- Conclusion

Preliminary for slide 2

Let me outline my talk. In introduction I will try to explain our general motivation - why for degenerate atomic gas one can expect a specific behavior of the scattering process in comparing with an ordinary bulk medium. Then I briefly review a number of experimental papers where, indeed, such an unusual behavior in light scattering from an atomic sample existing in a Bose-Einstein-Condensate phase have been observed. Next, just in a few slides I will try to explain the theoretical formalism, which we use in our approach to the problem, and introduce its basic parameters. Then I present the results of our numerical simulations, which have been done for one-dimensional model of light scattering, where I emphasize the importance of the quantum coherent description of the considered matter state.

Quantum matter state (BEC) = Position indistinguishability of atoms = Q-degeneracy

$$i\hbar\frac{d}{dt}\hat{\Psi}(\mathbf{r},t) = -\left[\hat{H},\hat{\Psi}(\mathbf{r},t)\right]$$

Coherence, Order parameter

$$n(\mathbf{r},t) = \left\langle \hat{\Psi}^{+}(\mathbf{r},t)\hat{\Psi}(\mathbf{r},t)\right\rangle = \Xi^{*}(\mathbf{r},t)\cdot\Xi(\mathbf{r},t)$$

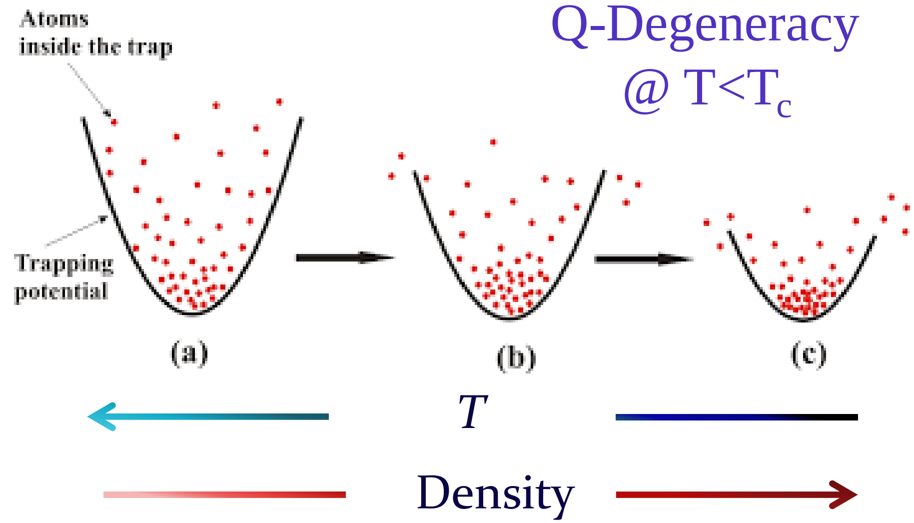
Weak interaction, GP equation

$$i\hbar\frac{\partial}{\partial t}\Xi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\Delta\,\Xi(\mathbf{r},t) + U_0\left[\Xi(\mathbf{r},t)\left|\Xi(\mathbf{r},t)\right|^2 - n_0\Xi(\mathbf{r},t)\right] + U_{ext}(\mathbf{r},t)$$

Internal int.: $U_0 = \int U(r)d^3r = \frac{4\pi\hbar^2\alpha}{m}$ where "*a*" is the scatt. length

Chemical potential.: $\mu_c \sim n_0 U_0$ GP required: $\mu_c \le \frac{\hbar^2 k_0^2}{2m} << \hbar\gamma$

Q-Degeneracy @ T<T_c



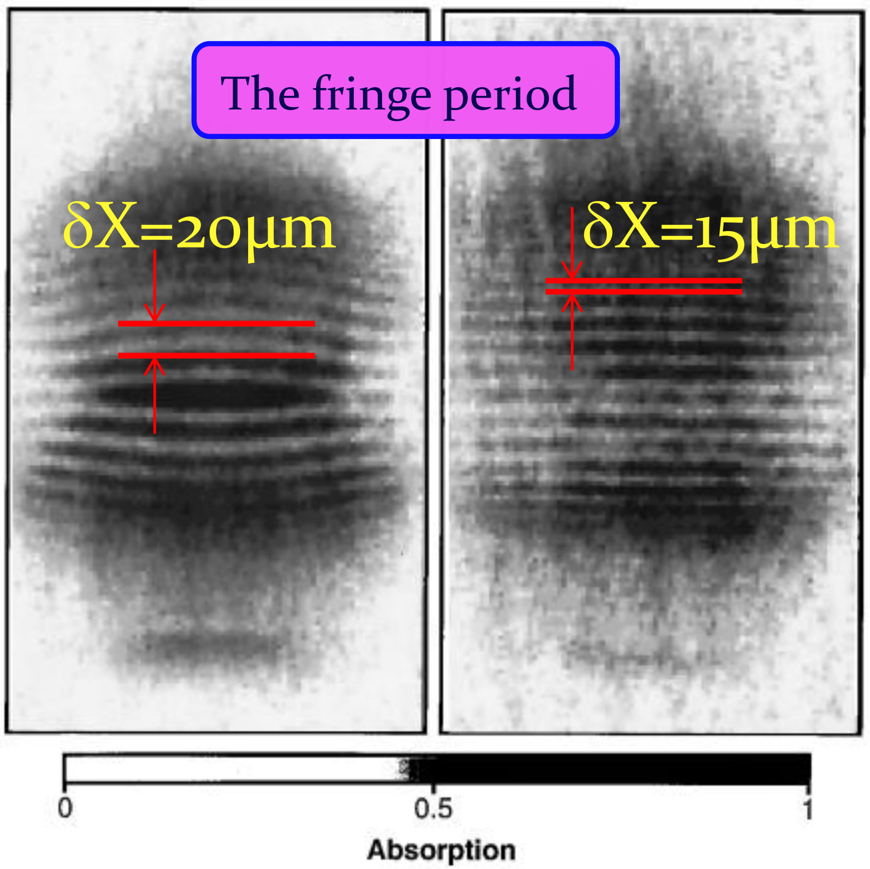
Preliminary for slide 3

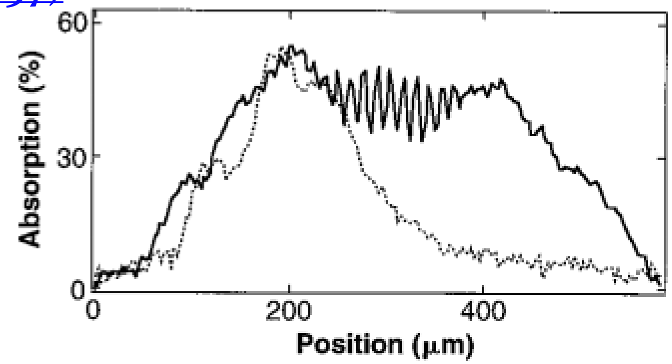
With lowering temperature an atomic gas transforms to the quantum degenerate state namely Bose-Einstein condensate phase. In such state the system demonstrates strong coherent behavior described by its matter wave function (order parameter). The order parameter obeys the so called Cross-Pitaevskii equation and parameterized by a combine interaction constant U-nil. In thermo-equilibrium this can be equivalently described by a chemical potential, which in the model is assumed to be smaller than the typical energy scale expressed by the recoil limit.

It can be expectable that such unusual properties of the matter state can lead to certain peculiarities in description of the light scattering process. In my presentation I will try to show how the scattering process on atomic system is modified in conditions of the quantum degeneracy.

Experimental evidence of coherent matter waves

M.R. Andrews, W. Ketterle, et al., Science 275, 637 (1997)





Time-of-flight images for a single and double condensate

Raw-data images of interference pattern of two expanding condensates for two different powers of the argon ion laser-light sheet

$$\Xi(\mathbf{r},t) = \Xi_+(\mathbf{r},t) + \Xi_-(\mathbf{r},t) \sim \Xi_+(z)\exp[i\Delta qz] + \Xi_-(z)\exp[-i\Delta qz]$$

Derivative of OP phase ~ condensate speed!

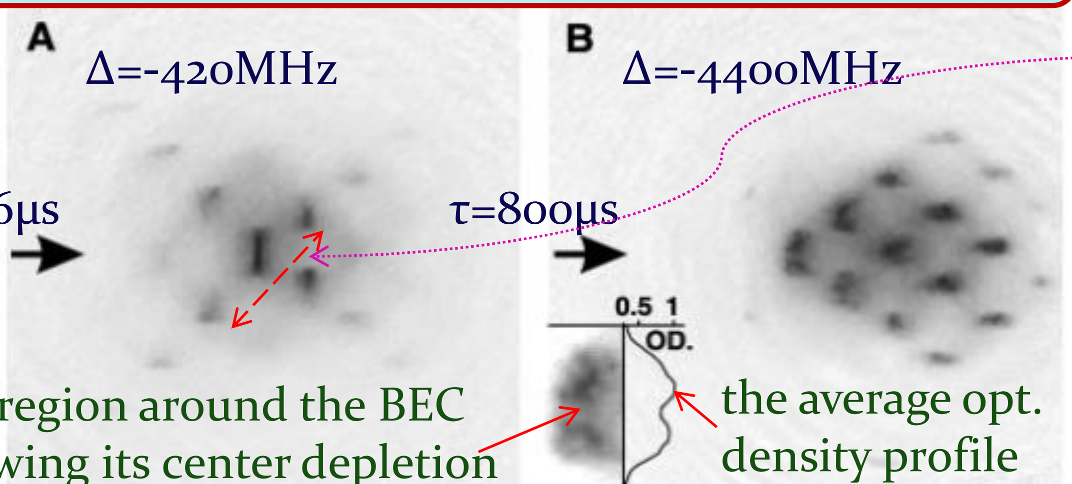
Preliminary for slide 4

In some experimental conditions, which are quite ordinary for any setup, BEC can be fractured on several fragment counter-propagating each other in their central mass reference frame. The matter wave interference creates strong density oscillation with a period parameterized by a relative speed of the BEC fragments. Such density oscillations can be strong and have quite short spatial period even in the case of dilute atomic sample with a small density of atoms in a volume scaled by cubic of its radiation wavelength. And as we will see a bit later in the last part of my talk this can initiate a strong Bragg-type scattering from the atomic sample.

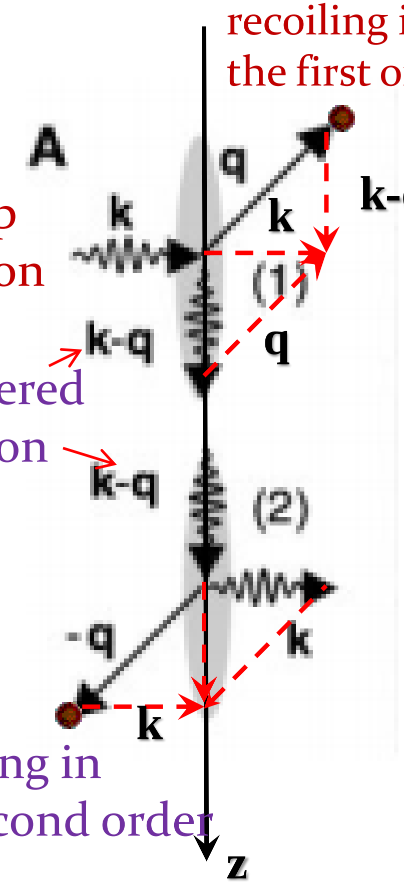
Light scattering from a BEC (I)

D. Schneble et al, Science 300, 475 (2003)

Superradiant scattering of a laser beam from a BEC in the **short** pulse **(A)** and **long** pulse **(B)**



Momentum transfer



Polariton = superposed light and matter waves

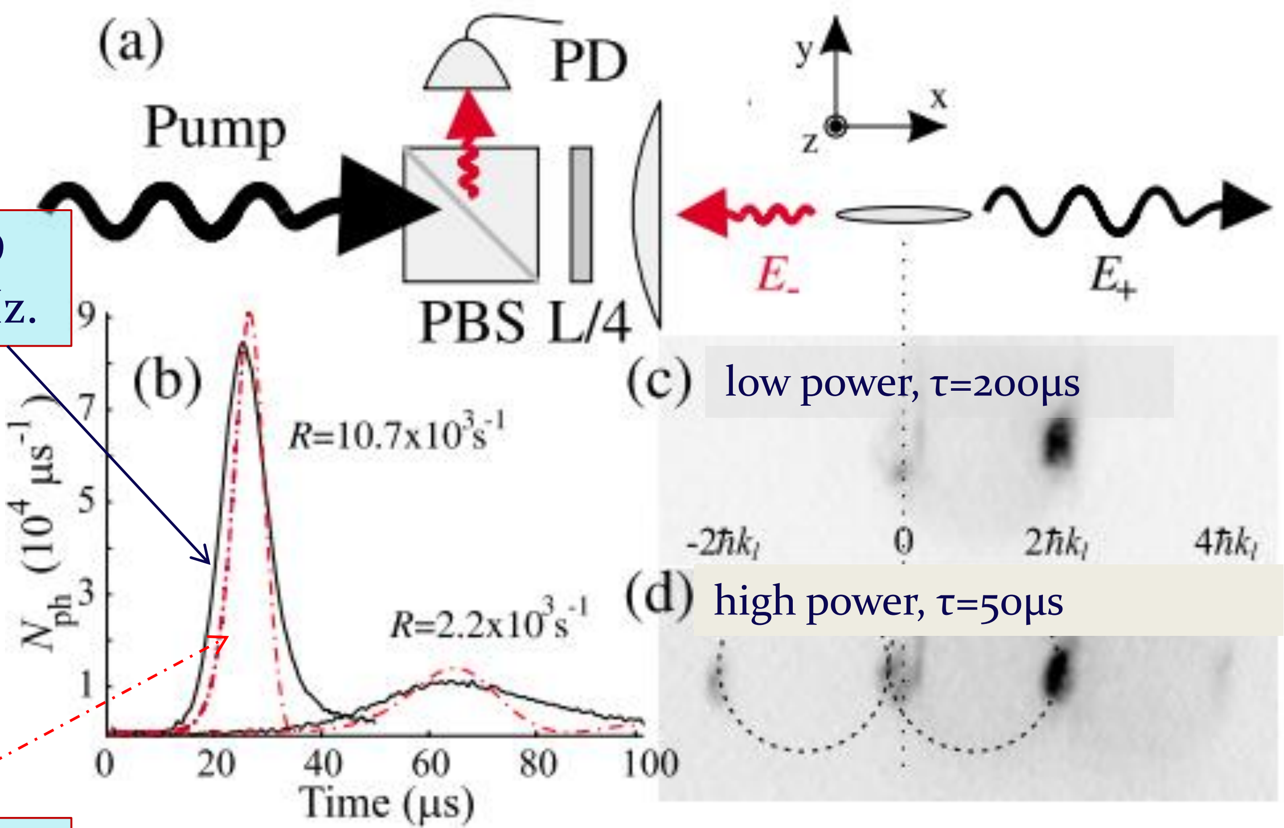
VP et al : J. Phys. 769 (2016) 012045 arXiv:1602.07562

Preliminary for slide 5

Such a fractured structure of BEC has been observed in a number of experiments. The optical excitation of the condensate by impinging photon[s] is expectedly converted to the polariton mode and escapes the sample, as an emitted photon, exactly in the direction of the polariton propagation. The polariton mode can be described by a Feynman-type graph equation, shown in the bottom of the slide. In accordance with the momentum conservation law the BEC fragments get kicks along its longest internal direction. In experiment by Schnebble et al this mechanism has created quite non-trivial “honeycomb” structure of the quantum matter in the entire scattering process.

Light scattering from a BEC (II)

A. Hilliard et al, Phys. Rev. A 78, 051403 (R) (2008)



Traces from PD at δ=-2π·2.6GHz.

Simulations

Absorption images of the atomic distributions.

Preliminary for slide 6

Similar BEC fragmentation has been independently observed in experiment by Hilliard et al. In that experiment the excitation light pulse consisting of many photons was directed along the longest longitudinal scale of the BEC sample. The polariton wave was created in the longitudinal direction and the BEC fragments was eventually ordered as a spatial lattice parameterized by different linear momenta. This mechanism of self organization in the quantum matter system can be understood as a certain kinematic entanglement of the spatially structured BEC.

Theoretical framework: *T*-matrix formalism

$$d\sigma_{i\to f} = \frac{\mathcal{V}^2 \omega'^2}{\hbar^2 c^4 (2\pi)^2} \left| T_{g'\mathbf{e}'\mathbf{k}';\, g\mathbf{e}\mathbf{k}}(E_i + i\,0) \right|^2 d\Omega$$

$$\hat{T}(E) = \hat{V} + \hat{V}\frac{1}{E - \hat{H}}\hat{V}$$

$$\hat{V} = -\sum_n \int d^3r \left[d^{\mu}_{nm} \hat{E}_{\mu}(\mathbf{r}) \hat{\Psi}^{\dagger}_n(\mathbf{r}) \hat{\Psi}_m(\mathbf{r}) + h.c. \right]$$

$$T_{fi}(E) = \frac{2\pi\hbar(\omega'\omega)^{1/2}}{\mathcal{V}} \iint d^3r'\, d^3r \sum_{n',n}$$

$$\times \;\; (\mathbf{d}\mathbf{e}')^*_{n'0} (\mathbf{d}\mathbf{e})_{0n} \mathbf{e}^{-i\mathbf{k}'\mathbf{r}' + i\mathbf{k}\mathbf{r}}\, \Xi^*(\mathbf{r}')\, \Xi(\mathbf{r})$$

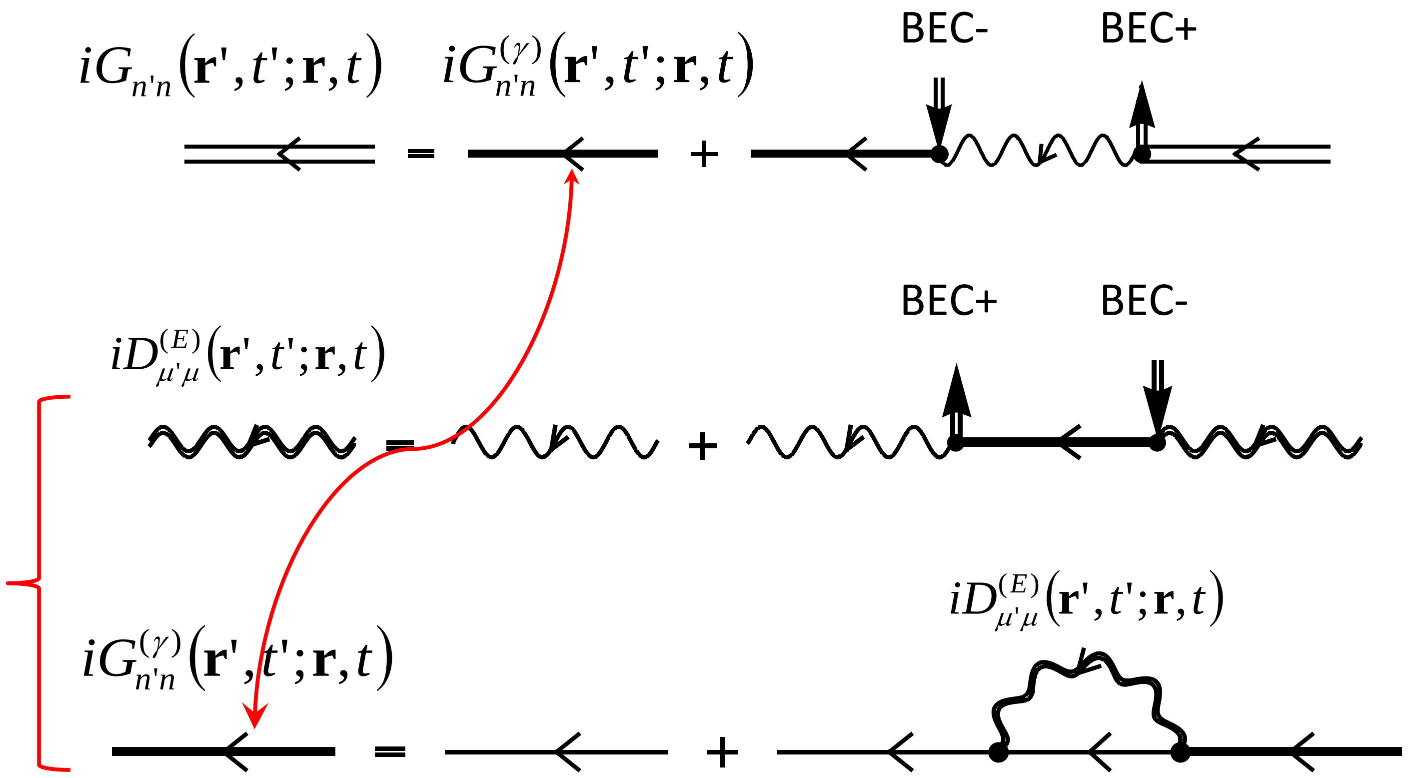
$$\times \;\; \left(-\frac{i}{\hbar}\right) \int_0^{\infty} dt\; \mathbf{e}^{\frac{i}{\hbar}(E - E_0^{N-1} + i0)t}\; i\, G_{n'n}(\mathbf{r}', t; \mathbf{r}, 0)$$

The subject of calculation

Preliminary for slide 7

Let me know turn to the theoretical description of the basic process of a single-photon scattering. In the quantum posed scattering theory the cross section for the process should be expressed by T-matrix, which is given by the combination of the interaction operator and the resolvent operator of the system Hamiltonian. In the second quantized formalism the matrix element is given by the following integral expansion shown in the bottom of this slide. The key element is the Green's function for an optical excitation propagating through the sample. This can be addressed as a polariton propagator. The propagator contributes into the T-matrix being overlapped with the order parameter at the points where the polariton was created and anihilated.

Diagram expansion of the Green's function



Preliminary for slide 8

The polariton propagator can be obtained via its perturbation theory expansion based on the Feynman diagram method. The complete propagator is given by the integral equation, which is expressed by the upper graph. To construct this equation in its closed form, we have to initially find the contributions of the incomplete propagator and photon's Green's function in the medium. (Animation here)

These functions are defined by the two lower graph equations and both physically describe the process of incoherent (spontaneous) scattering. Since such scattering transfer an atom out of the condensate phase the structure of this function is similar to if the scattering process were developing in an disordered atomic gas of the same density.

Eventually we can construct the closed integral equation for the complete propagator, which is quite difficult even for numerical solution because of very complicated structure of its self-energy part. But the solution can be found in a limit of an infinite and homogeneous medium (without scattering), and in a model of one dimensional configuration, as I explain in the next slide.

1D-sacttering problem

T- and *S*- matrices:

$$T_{i'i}(E) = \frac{2\pi\omega}{\mathcal{L}} \iint dz'\, dz \sum_{n',n} (\mathbf{de})^*_{n'0} (\mathbf{de})_{0n}$$

$$\times \mathbf{e}^{-ik'z'+ikz}\, \Xi^*(z')\, \Xi(z)\, G_{n'n}(z', z; E - E_0^{N-1})$$

$$S_{i'i} \;=\; \delta_{i'i} - i\, \frac{\mathcal{L}}{\hbar c}\, T_{i'i}(E_i + i0)$$

Transmission, Reflection, Losses:

$$\mathrm{T}(\omega) \;=\; \left|S_{i'i}\right|^2\Big|_{k'=k>0}$$

$$\mathrm{R}(\omega) \;=\; \left|S_{i'i}\right|^2\Big|_{k'=-k<0}$$

$$\mathrm{L}(\omega) \;=\; 1 - \mathrm{T}(\omega) - \mathrm{R}(\omega)$$

The Green's function:

$$G_{nn'}(z, z'; E) = \delta_{nn'}\, G(z, z'; E)$$

$$G(z, z'; E) \;=\; \frac{1}{L} \sum_{s,s'} \mathrm{e}^{ik_s z - ik_{s'} z'} G_{ss'}(E)$$

$$\left[\omega - \tilde{\omega}_0 + \frac{\hbar\, k_s^2}{2\mathrm{m_A}} + \frac{4\pi}{3\hbar}\, n_0\, d_0^2 + \frac{i}{2}\sqrt{\epsilon\,(\omega)}\;\gamma\right] G_{ss'}(\omega)$$

$$-\sum_{s''} \Sigma^{(c)}_{ss''}(\omega) G_{s''s'}(\omega) = \delta_{ss'}$$

<= numerically solved

Preliminary for slide 9

In one-dimensional description of the scattering problem instead of cross-section we have to introduce the coefficients of transmission, reflection and losses, shown in the right. These quantities are given by the S-matrix, which is straightforwardly expressed by the T-matrix, as shown in the left. The order parameter as well as the Green’s function now depend only on z-coordinate. The integral equation can be transformed to a system for infinite set of algebraic equations via spatial Fourier representation on the sample segment. This equation is shown in the bottom of the slide and eventually can be numerically solved.

This one-dimensional model gives us resource for evaluation of the forward and backward single-photon scattering, which we can compare with predictions of the conventional macroscopic Maxwell theory for the similar process developed in an disordered atomic gas.

Microscopic approach vs. Maxwell theory:
Uniform profile of the OP

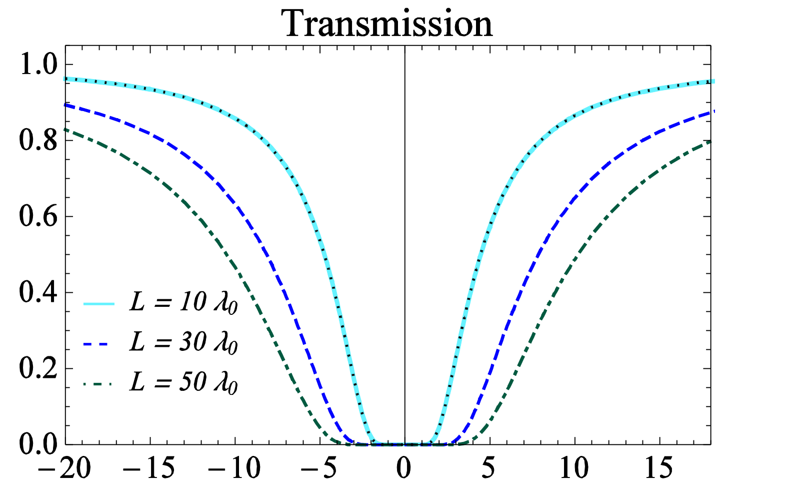
Maxwell theory:

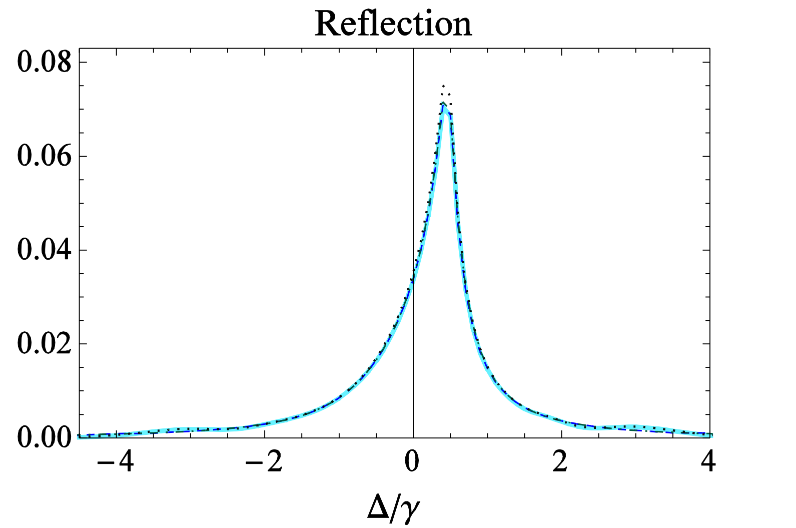
$$\mathrm{T}(\omega) = \left| \frac{2\sqrt{\epsilon(\omega)}}{2\sqrt{\epsilon(\omega)}\cos\psi(\omega) - i(1+\epsilon(\omega))\sin\psi(\omega)} \right|^2$$

$$\mathrm{R}(\omega) = \left| \frac{\sin\left[\psi(\omega)\right]}{\sin\left[\psi(\omega) - i\ln\frac{1-\sqrt{\epsilon(\omega)}}{1+\sqrt{\epsilon(\omega)}}\right]} \right|^2$$

$$\Xi(\mathbf{r}) = \sqrt{n_0}$$

$$\psi(\omega) = L\sqrt{\epsilon(\omega)}\omega/c.$$





Preliminary for slide 10

In this slide we present the results of our numerical simulations , which were done for uniform profile of the order parameter that means for a homogeneous distribution of atoms in a sample. We compare the results with prediction of the conventional macroscopic Maxwell theory, which are presented by the solutions shown in the left. For the latter case the parameter of dielectric permittivity can be found in the self-consistent model for a disordered atomic gas consisting of the same atoms as BEC.

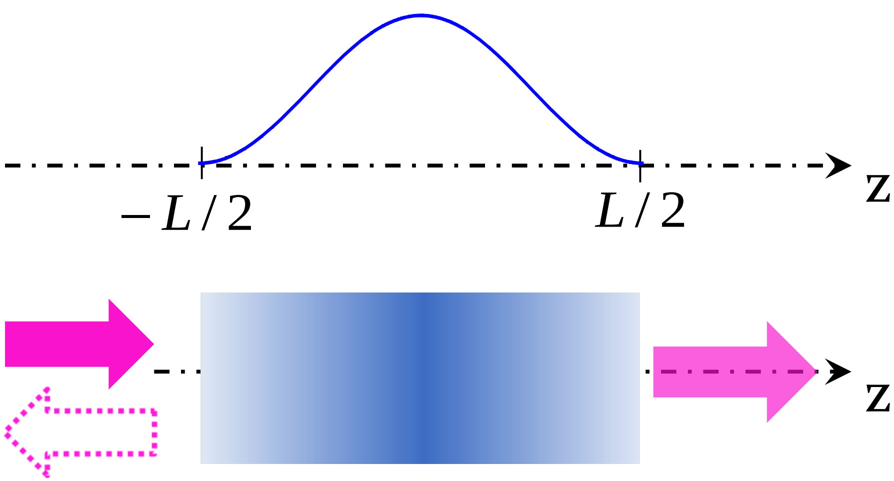
We obtained excellent “point by point” coincidence between the results of both the calculation schemes! The density averaging is insensitive to the type of distribution for either coherent matter wave or for a system of disordered point-like scatterers. Nevertheless, we would like to stress that the plotted dependencies result as the solutions of principally different (!) equations.

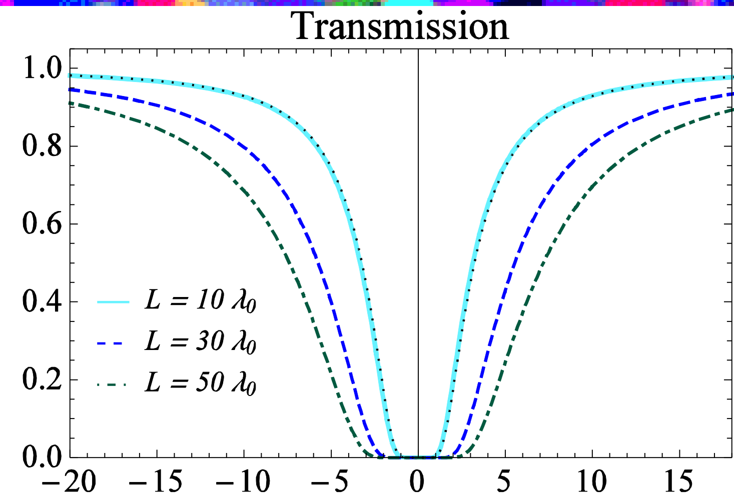
Microscopic approach vs Maxwell theory:

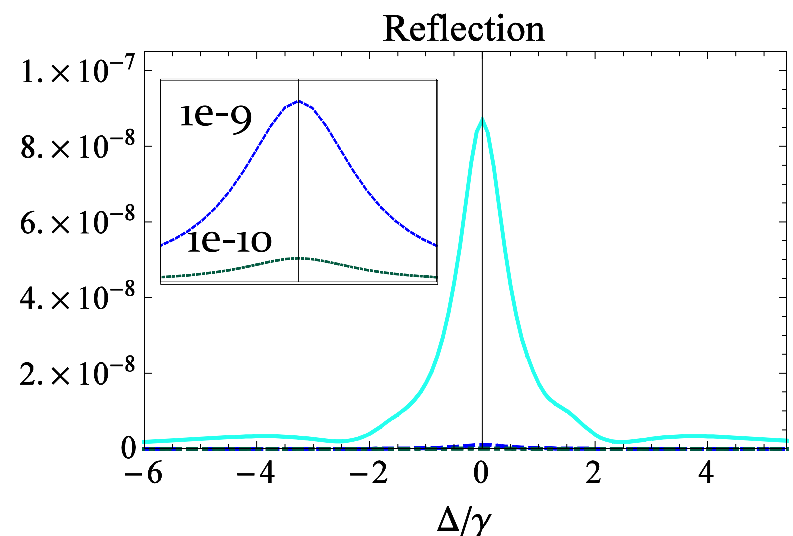
trigonometric profile of the OP

$$\Xi(z) = \sqrt{n_0}\,\cos\left(\frac{\pi z}{L}\right)$$

$$n(z) = \left|\Xi(z)\right|^2$$







Preliminary for slide 11

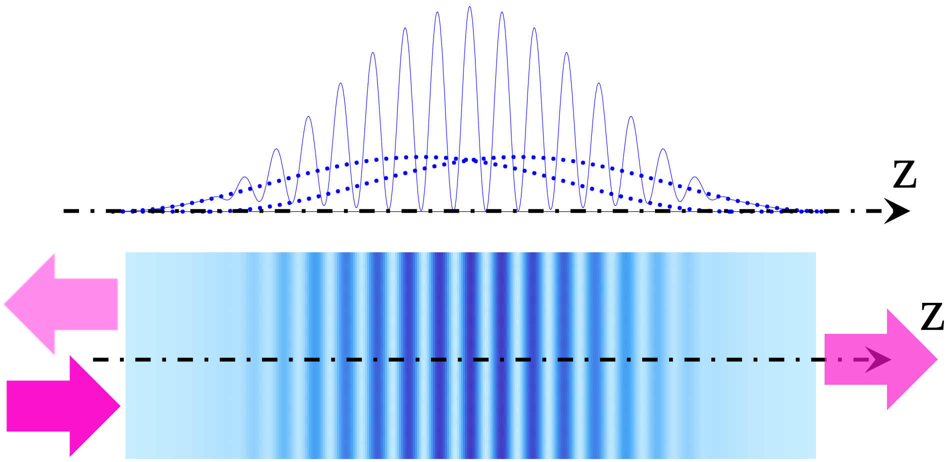
The last point can be ultimately verified via a round of independent calculations performed for the order parameter of a trigonometric profile. Both the uniform and cosine-type shapes of the order parameter can be accepted as an example of the Thomas-Fermi solution of the Gross-Pitaevskii equation. Again we have obtained excellent coincidence between the microscopic and macroscopic calculation approaches. The intensity of reflected light is much weaker in the case of smoothed sample bounds since we have no coherent scattering from a sample edges in this case.

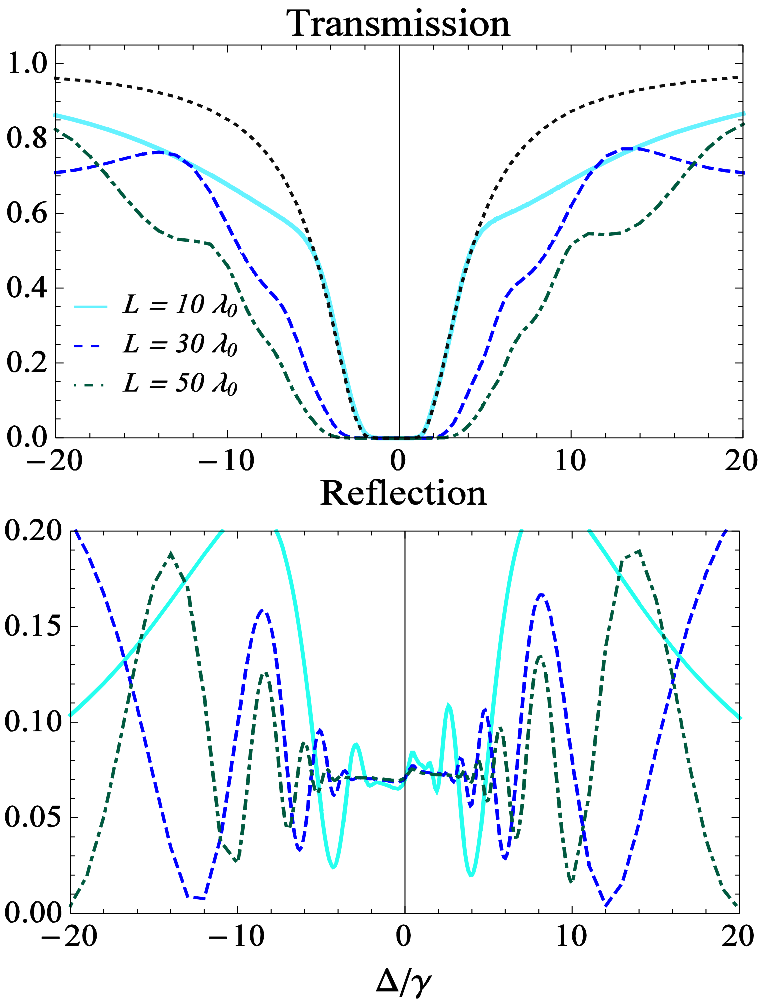
Microscopic approach vs Maxwell theory:

Interference of two BEC fragments

$$\Xi(z) = \Xi_+(z) + \Xi_-(z)$$

$$\Xi_\pm(z) \sim \cos\left(\frac{\pi z}{L}\right)\exp[\pm i\Delta q z]$$





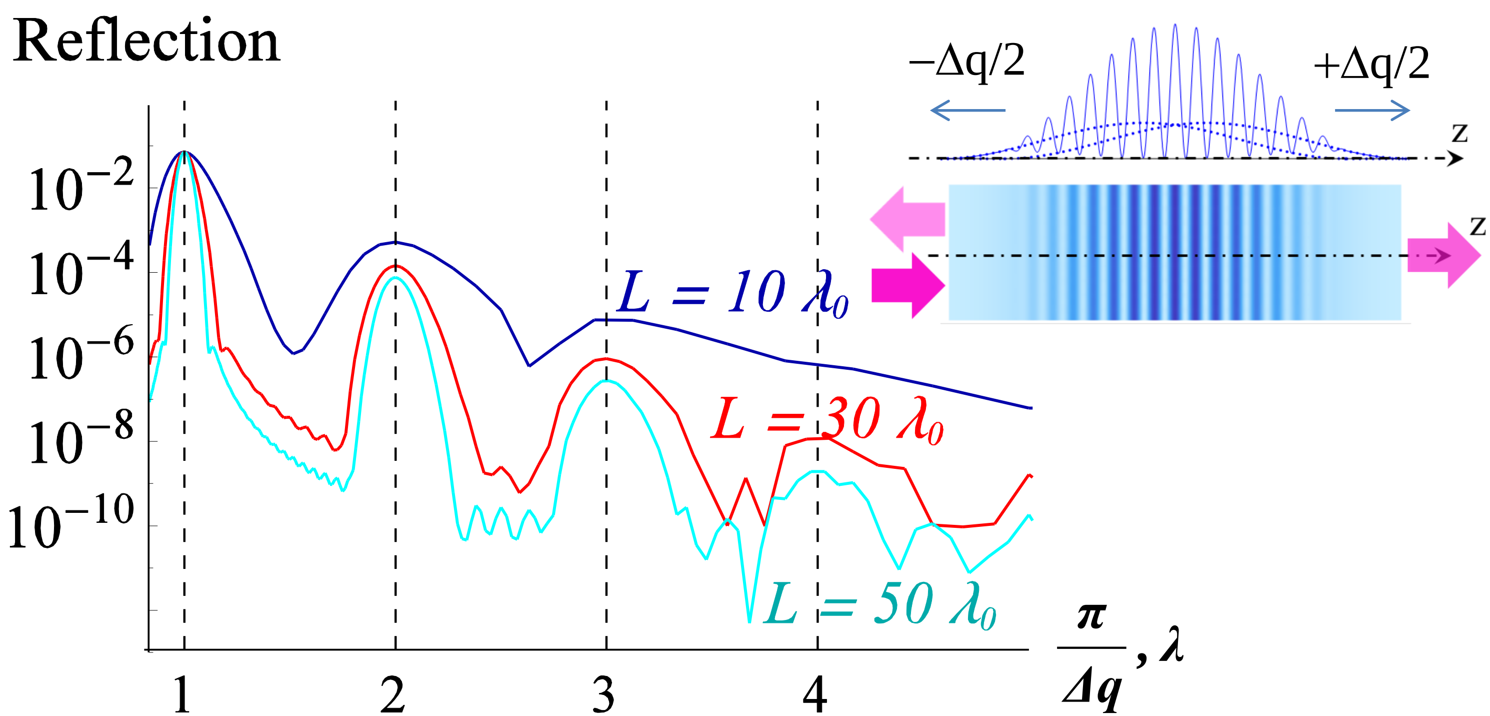
Preliminary for slide 12

The situation has been dramatically changed once we consider it for two interfering BEC fragments, as I have pointed out in the beginning of my talk. Now we can observe quite strong scattering in forward as well as in the backward direction initiated by the spatial modulation of the order parameter. This type of scattering can be associated with a mechanism of the Bragg-type diffraction on the spatially oscillating density.

Based on predictions of the conventional Maxwell theory, the transmission should have a broad absorption domain, shown by the dotted curve, and reflection should be negligible. But in reality we can observe strong enhancement of the reflection as well as the strongly modified non-monotonic transmission spectra.

Bragg scattering in the backward direction

$$\Xi(z)=\Xi_+(z)+\Xi_-(z)=\sqrt{\frac{n_0}{2}}\cos\left(\frac{\pi z}{L}\right)e^{i\Delta qz}+\sqrt{\frac{n_0}{2}}\cos\left(\frac{\pi z}{L}\right)e^{-i\Delta qz}$$



Preliminary for slide 13

The crucial role of the Bragg-type diffraction in the scattering can be ultimately followed once we consider the reflection as function of the matter-waves relative wavenumber “Δq”. This parameter is associated with the relative motion of the condensate fragments in its central mass reference frame. We can clear track the existence of local maxima at the points where “Δq” is scaled by the radiation wavelength. In those case when the condensate fragments move with the relative speed, expressed by the recoil-limit linear momentum, the density oscillations lead to the strongest scattering.

Conclusion:

- We have developed the microscopic scattering approach for description of a single-photon scattering from a quantum degenerate bosonic atomic gas
- The crucial point is that in the considered case the atomic medium represents a coherent matter wave strongly rejecting its classical interpretation
- The scattering equation has been solved in one-dimensional geometry and the results have been compared with predictions of the conventional macroscopic Maxwell theory for a disordered atomic gas
- The light scattering is strongly enhanced by mechanism of Bragg diffraction on a density (order parameter) oscillations caused by internal motion in the condensate

Thank you for your attention!

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